

1 Indices

1.1 Multiplication and Division

1. Calculate the following mentally:

- | | | |
|-------------------------|--------------------------|------------------------|
| (a) $14 + 6 + 9$ | (b) $14 + 21 + 9$ | (c) $31 + 16 + 9$ |
| (d) $25 + 28 + 15$ | (e) $67 + 52 + 33$ | (f) $123 + 66 + 77$ |
| (g) $28 + 22 + 41 + 59$ | (h) $49 + 51 + 101 + 99$ | (i) $7 + 25 + 13 + 75$ |
| (j) $24 + 12 - 14$ | (k) $52 - 21$ | (l) $37 - 18$ |
| (m) $25 - 12 + 7$ | (n) $72 + 21 - 37$ | (o) $112 - 27$ |

2. Calculate the following mentally:

- | | | |
|------------------|------------------|------------------|
| (a) 5×7 | (b) 9×7 | (c) 8×8 |
| (d) 6×8 | (e) 7×4 | (f) 7×8 |
| (g) 8×9 | (h) 5×8 | (i) 9×6 |
| (j) $72 \div 8$ | (k) $35 \div 7$ | (l) $81 \div 9$ |
| (m) $42 \div 6$ | (n) $63 \div 9$ | (o) $49 \div 7$ |
| (p) $56 \div 8$ | (q) $45 \div 5$ | (r) $48 \div 6$ |

3. Simplify the following expressions *without* using a calculator.

- | | | |
|---------------------------------|----------------------------|--------------------------------------|
| (a) $37 + (5 \times 6)$ | (b) $(7 \times 7) - 28$ | (c) $96 - (12 \times 5)$ |
| (d) $9 \times (3 + 5)$ | (e) $(15 + 6) \times 9$ | (f) $7 \times (69 - 65)$ |
| (g) $(48 \div 6) + 12$ | (h) $4 + (78 \div 6)$ | (i) $(72 \div 8) - 5$ |
| (j) $90 \div (5 + 4)$ | (k) $(27 + 33) \div 5$ | (l) $(92 - 29) \div 7$ |
| (m) $(54 \div 9) + (45 \div 3)$ | (n) $(63 \div 9) \times 5$ | (o) $(72 \div 6) \times (45 \div 9)$ |

4. Pens cost 12 p each. How much will 8 pens cost?

5. A crate of small lemonade bottles has 8 rows with 6 bottles in each row. How many bottles are there in the crate?

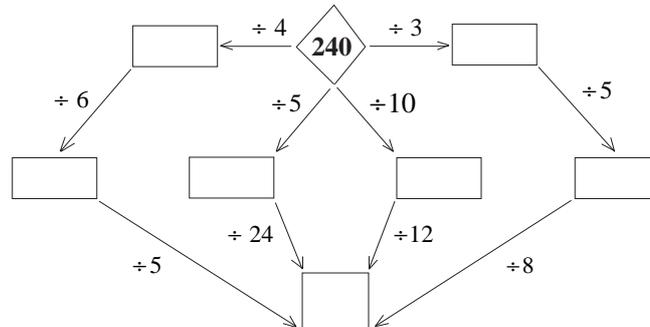
6. A chocolate bar has 5 rows with 4 chunks in each row. How many chunks are there in total?

7. In snooker, potting a RED counts as one point and a BLACK as seven points. What do you score if you pot 6 REDS and 6 BLACKS?

8. A syndicate of 7 teachers won £63 on the national lottery. How much does each teacher win?

9. There are 8 stamps in a booklet. How many stamps are there in total in 7 booklets?

10. There are 64 runners entered for 100 m races. If there are 8 runners in each race in the first round of heats, how many first round heats are there?
11. You need to divide a class of 35 pupils into five-a-side teams. How many teams will there be?
12. Copy and complete this diagram by filling in each box with the correct number.



13. Tony has a hot-dog stall. He uses sauce from a bottle which holds 224 ml. He puts about 7 ml of sauce on each hot-dog.
- (a) How many hot-dogs can he put sauce on from one bottle?
- (b) Tony buys the sauce in boxes of 24 bottles. One full bottle weighs 256 g. The empty box weighs 750 g.
What is the total weight of a box full of sauce bottles?
- (NEAB)
14. Write down three multiplication sums, each of which has an answer equal to 30. Do not use any number more than once.
- (LON)
15. Look at these numbers.
- 5, 10, 15, 20, 25, 30, 35, 40
- (a) Which number is a multiple of six?
- (b) Which number is a square number?
- (SEG)

1.2 Squares, Cubes, Square Roots and Cube Roots

1. Find, without using a calculator:

- | | | |
|------------|------------|-----------|
| (a) 5^3 | (b) 2^4 | (c) 3^3 |
| (d) 4^2 | (e) 7^2 | (f) 1^3 |
| (g) 2^3 | (h) 5^2 | (i) 9^2 |
| (j) 10^2 | (k) 10^3 | (l) 8^2 |
| (m) 7^3 | (n) 6^2 | (o) 4^3 |

2. Find, *without* using a calculator:

- | | | |
|---------------------|----------------------|------------------|
| (a) $\sqrt{4}$ | (b) $\sqrt[3]{27}$ | (c) $\sqrt{36}$ |
| (d) $\sqrt[3]{8}$ | (e) $\sqrt{81}$ | (f) $\sqrt{49}$ |
| (g) $\sqrt{100}$ | (h) $\sqrt[3]{64}$ | (i) $\sqrt{144}$ |
| (j) $\sqrt{64}$ | (k) $\sqrt{121}$ | (l) $\sqrt{25}$ |
| (m) $\sqrt[3]{125}$ | (n) $\sqrt[3]{1000}$ | (o) $\sqrt{169}$ |

3. Calculate, *with or without* a calculator,

- | | | |
|--------------------------------|--------------------------------------|-------------------------------------|
| (a) $2^3 - 3^2$ | (b) $4^2 - 2^4$ | (c) $5^3 + 5^2$ |
| (d) $1^2 + 2^2$ | (e) $1^2 + 2^2 + 3^2$ | (f) $9^2 - 7^2$ |
| (g) $\sqrt{4} + \sqrt{9}$ | (h) $\sqrt{36} \div \sqrt{4}$ | (i) $\sqrt[3]{8} \div \sqrt{16}$ |
| (j) $\sqrt{100} + \sqrt{25}$ | (k) $\sqrt{400} \div \sqrt[3]{125}$ | (l) $\sqrt{196} - \sqrt{64}$ |
| (m) $\sqrt{64} - \sqrt[3]{64}$ | (n) $\sqrt[3]{343} \times \sqrt{81}$ | (o) $\sqrt[4]{625} \times \sqrt{4}$ |

4. Find the next 5 terms in the sequence.

$$2, 2 \times 2, 2 \times 2 \times 2, 2 \times 2 \times 2 \times 2, \dots, \dots$$

How many terms are needed before the value reaches one million?

1.3 Index Notation

1. Write in a form using indices:

- | | |
|---|---|
| (a) $2 \times 2 \times 2 \times 2$ | (b) $3 \times 3 \times 3$ |
| (c) $6 \times 6 \times 6 \times 6 \times 6$ | (d) $7 \times 7 \times 7 \times 7 \times 7 \times 7$ |
| (e) $1 \times 1 \times 1 \times 1$ | (f) $2 \times 2 \times 2 \times 5 \times 5$ |
| (g) $17 \times 17 \times 17 \times 17$ | (h) $5 \times 5 \times 5 \times 6 \times 6 \times 6$ |
| (i) $3 \times 3 \times 7 \times 7 \times 7 \times 7 \times 7$ | (j) $2 \times 2 \times 3 \times 3 \times 3 \times 5 \times 5$ |
| (k) $5 \times 3 \times 3 \times 3 \times 5$ | (l) $11 \times 11 \times 11 \times 11 \times 13 \times 13$ |

2. Find the value of the following:

- | | | |
|------------|--------------|-----------|
| (a) 7^2 | (b) 3^3 | (c) 3^5 |
| (d) 2^8 | (e) 7^0 | (f) 5^3 |
| (g) 4^3 | (h) 6^3 | (i) 1^7 |
| (j) 10^6 | (k) 2^{10} | (l) 3^6 |

3. Simplify each of the following, leaving your answer in index notation.

- (a) $2^7 \times 2^5$ (b) $3^2 \times 3^4 \times 3^6$ (c) $5^2 \times 5^2 \times 5^2$
 (d) $4^3 \times 4^7$ (e) $7^2 \times 7^4 \times 7^3$ (f) $2^1 \times 2^5 \times 2^2 \times 2^1$
 (g) $2^3 \times 2^3 \times 5^1 \times 5^3$ (h) $3^2 \times 3^3 \times 4^5 \times 4^2 \times 3^1$ (i) $5^2 \times 5^4 \times 5^7$

4. Simplify each of the following, leaving your answer in index notation.

- (a) $3^4 \div 3^3$ (b) $7^5 \div 7^2$ (c) $(2^3 \times 2^5) \div 2^8$
 (d) $8^{12} \div 8^7$ (e) $(4^3 \times 4^3) \div 4^3$ (f) $2^6 \div (2^3 \times 2^2)$
 (g) $(9^{10} \times 9^4) \div 9^6$ (h) $(6^{11} \div 6^{10}) \times 6^3$ (i) $(10^9 \times 10^9) \div 10^{16}$
 (j) $(4^7 \div 4^3) \times 4^2$ (k) $(2^1 \times 2^2 \times 2^3) \div 2^4$ (l) $6^8 \div (6^1 \times 6^2 \times 6^3)$

5. Express each of the following numbers as a number to a power, e.g. $256 = 2^8$.

- (a) 1024 (b) 243 (c) 125 (d) 216
 (e) 512 (f) 169 (g) 343 (h) 1000
 (i) 625 (j) 2048 (k) 289 (l) 1331

6. Fill in the missing numbers.

- (a) $(2^3)^2 = 2^?$ (b) $(3^3)^3 = 3^?$ (c) $(5^?)^3 = 5^{12}$
 (d) $(2^4)^? = 2^8$ (e) $(5^3)^2 = 5^?$ (f) $(4^3)^? = 4^{15}$
 (g) $(10^3)^? = 10^9$ (h) $(7^4)^2 = 7^?$ (i) $(2^?)^6 = 2^{12}$
 (j) $(3^?)^7 = 3^{21}$ (k) $(2^4)^? = 2^{16}$ (l) $(6^?)^4 = 6^{20}$

7. Simplify the following expressions, leaving your answers in index notation.

- (a) $a^4 \times a^3$ (b) $x^5 \div x^2$ (c) $(b^4 \times b^3) \div b^5$
 (d) $a^4 \div a^3$ (e) $x^4 \times x^5$ (f) $(x^4 \times x^5)^2$
 (g) $(a^5 \div a^2) \times a$ (h) $(a^3)^2 \times (a^2)^3$ (i) $(x^2 \times x^3)^2 \div x^4$
 (j) $(b^4 \div b^2)^3$ (k) $(b^4)^3 \div (b^2)^3$ (l) $[a^4 \times (a^2)^3] \div a^8$
 (m) $\frac{x^7 \times x^2}{x^4}$ (n) $\frac{a^4 \times (a^2)^2}{a^8}$ (o) $\frac{x^5}{x^2 \times x^2}$

8. (a) Fill in the missing number.

$$2^4 \times 2^? = 2^{10}$$

- (b) 2^{10} is approximately equal to 1000.

1 000 000 is approximately equal to $2^?$

(SEG)

1.4 Factors

1. Which of the following numbers are divisible by 2, 4 or 5?

- | | | | |
|---------|---------|----------|----------|
| (a) 10 | (b) 24 | (c) 60 | (d) 108 |
| (e) 135 | (f) 189 | (g) 240 | (h) 315 |
| (i) 648 | (j) 756 | (k) 1024 | (l) 2410 |

2. Which of the following numbers are divisible by 3, 9 or 11?

- | | | | |
|---------|---------|----------|----------|
| (a) 18 | (b) 72 | (c) 126 | (d) 441 |
| (e) 649 | (f) 825 | (g) 1419 | (h) 9372 |

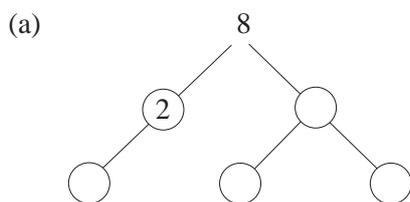
3. List all factors of these numbers.

- | | | | |
|--------|---------|--------|--------|
| (a) 20 | (b) 24 | (c) 32 | (d) 84 |
| (e) 60 | (f) 102 | (g) 81 | (h) 98 |

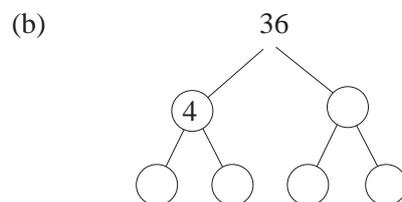
4. Copy and fill in the missing factors in each of the following.

- | | | |
|---------------------------------|---------------------------------|--|
| (a) $35 = 5 \times ?$ | (b) $77 = 7 \times ?$ | (c) $96 = 8 \times ?$ |
| (d) $42 = 2 \times 3 \times ?$ | (e) $66 = 2 \times 3 \times ?$ | (f) $72 = 3 \times 4 \times ?$ |
| (g) $104 = 2 \times 4 \times ?$ | (h) $125 = 5 \times 5 \times ?$ | (i) $220 = 2 \times 2 \times 5 \times ?$ |

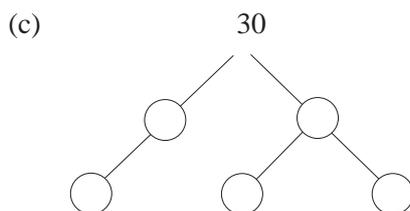
5. Copy and complete the following factor trees.



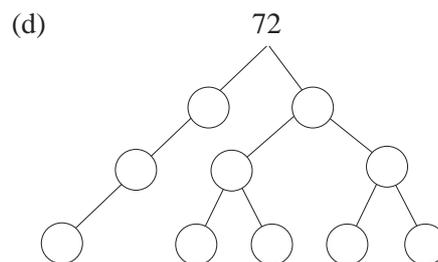
$$8 = ? \times ? \times ?$$



$$36 = ? \times ? \times ? \times ?$$



$$30 = ? \times ? \times ?$$



$$72 = ? \times ? \times ? \times ? \times ? \times ?$$

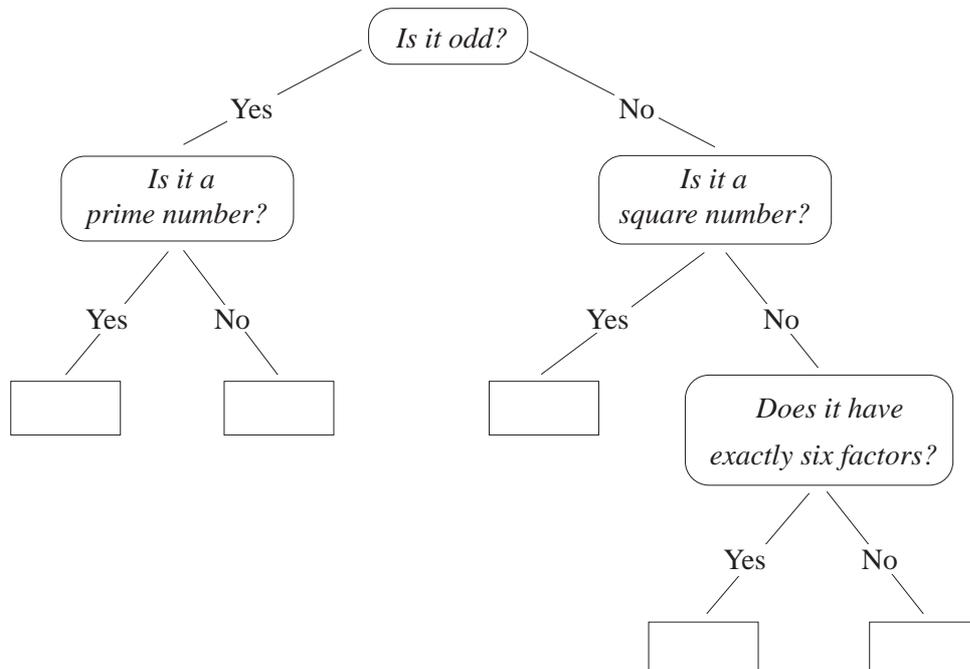
1.5 Prime Factors

- Which of the numbers 2, 3, 5, 7, 11, 13 are prime factors of the following numbers?

(a) 189	(b) 264	(c) 490	(d) 720
(e) 165	(f) 288	(g) 2873	(h) 2310
(i) 3640	(j) 6006	(k) 1925	(l) 1815
- Use the *Sieve of Eratosthenes* to find all prime numbers between 100 and 200.
- Express the following numbers as products of prime numbers.

(a) 150	(b) 60	(c) 72	(d) 144
(e) 315	(f) 210	(g) 284	(h) 180
(i) 270	(j) 231	(k) 306	(l) 500
(m) 702	(n) 3234	(o) 8008	(p) 8190
- Find the highest common factor of the following.

(a) 16 and 24	(b) 45 and 63	(c) 56 and 70
(d) 90 and 126	(e) 42, 66 and 78	(f) 84, 98 and 154
(g) 189 and 84	(h) 315 and 720	(i) 616 and 392
(j) 560, 140 and 224	(k) 132, 156 and 180	(l) 525, 1400 and 315
- (a) Copy and put {9, 17, 28, 30} into the correct boxes.



- (b) Write down a number that could go into the empty box.

(SEG)

6. The speed of light is approximately 300 000 km/s.
- Express this speed in m/s in standard form.
 - The speed of sound is 300 m/s. How many times more than the speed of sound is the speed of light?
Give your answer in standard form.
7. The population of Singapore in a given year was 2.5×10^6 .
Its total land area is estimated as 618 km². What was the average population per square km in that year?
Give your answer in standard form, correct to 2 significant figures.

1.8 Calculations with Standard Form

1. Without using a calculator, work out the following calculations.
Express your answer in standard form.
- | | |
|--|---|
| (a) $(2.8 \times 10^4) + (3 \times 10^3)$ | (b) $(2.8 \times 10^4) - (3 \times 10^3)$ |
| (c) $(6.3 \times 10^3) + (5.37 \times 10^4)$ | (d) $(9.7 \times 10^2) + (0.3 \times 10^3)$ |
| (e) $(4 \times 10^3) \times (2 \times 10^2)$ | (f) $(5 \times 10^4) \times (3 \times 10^2)$ |
| (g) $(8 \times 10^4) \div (4 \times 10^2)$ | (h) $(1.5 \times 10^3) \times (2 \times 10^6)$ |
| (i) $(6.5 \times 10^2) \times (2 \times 10^4)$ | (j) $(9 \times 10^2) \div (2 \times 10^3)$ |
| (k) $(6.4 \times 10^6) \div (1.6 \times 10^3)$ | (l) $(8.4 \times 10^5) \div (2.1 \times 10^3)$ |
| (m) $(2.5 \times 10^6) \times (4 \times 10^{-4})$ | (n) $(3.4 \times 10^{-4}) \times (2 \times 10^3)$ |
| (o) $(5 \times 10^{-3}) \times (2 \times 10^{-2})$ | (p) $(2.4 \times 10^{-2}) \times (5 \times 10^2)$ |
2. Write $(4 \times 10^{-4}) + (8 \times 10^{-3})$ as a single number expressed in standard form.
3. Given that $x = 2 \times 10^{-3}$ and $y = 7 \times 10^{-4}$, express $x + 8y$ in standard form.
4. Express $10^{-6} - (2.5 \times 10^{-7})$ in standard form.
5. Given that $x = 3.2 \times 10^6$ and $y = 5 \times 10^7$, express in standard form:
- | | | | |
|----------|-------------------|-----------|----------------------------------|
| (a) xy | (b) $\frac{x}{y}$ | (c) y^2 | (d) $\left(\frac{x}{y}\right)^2$ |
|----------|-------------------|-----------|----------------------------------|
6. In the formula $R = \frac{M}{EI}$, substitute
 $M = 6 \times 10^4$, $E = 4.5 \times 10^8$, $I = 4 \times 10^2$
 and evaluate R , giving your answer in standard form.

7. The radius of a circular micro-organism is 2.8×10^{-7} cm.
Calculate the circumference and area of the micro-organism, giving your answer in standard form.
- 8.* Evaluate each of the following expressions, giving your answer in standard form.
- (a) $6.39^3 \times 7.8^4$ (b) $16.3^4 - 3.65^6$
 (c) $7.81 \times 10^3 - 0.13^{-2}$ (d) $6.3 \times 10^{14} \times 8.91^9$
 (e) $9.94 \times 10^{12} \div 23.5^4$ (f) $\sqrt{3.62 \times 10^{-4}}$
 (g) $4.5 \times 10^2 \times \sqrt{7.26 \times 10^{-6}}$ (h) $\frac{5.21 \times 10^{-4}}{\sqrt{2.26 \times 10^4}}$
 (i) $\sqrt{\frac{7.2 \times 10^8}{4.31 \times 10^6}}$ (j) $\left[(4.12 \times 10^3) \div (6.25 \times 10^2) \right]^{\frac{3}{2}}$
9. Given that $\frac{15.3 \times 12.4}{5.1 \times 31} = 1.2$, *without* using a calculator, find the value of

$$\frac{15.3 \times 1.24}{51 \times 3.1}$$
 and express it in standard form.
- 10.* If $x = 3.6 \times 10^{-2}$, evaluate and express your answer in standard form:
 (a) $4x^2 + x$ (b) $\sqrt{x + 1}$
11. Given that $x = 5 \times 10^5$, find the value of each of the following, giving your answer in standard form.
 (a) $5x$ (b) x^2 (c) $\frac{2}{x}$ (d) $\frac{3500}{x^2}$
12. Work out $4 \times 10^8 - 4 \times 10^6$. Give your answer in standard form. (LON)
13. Saturn is approximately 1.43×10^9 km from the Sun.
 Venus is approximately 1.08×10^8 km from the Sun.
 How much further from the Sun is Saturn than Venus?
 Give your answer in standard form. (SEG)
14. A light year is the distance travelled by light in 365 days.
 The speed of light is 3.9×10^5 kilometres per second.
 (a) Calculate the number of kilometres in one light year.
 Give your answer in standard form.

- (b) The distance to the nearest star is 4.0×10^{13} kilometres.
How many light years is this?
- (c) One kilometre = 0.625 miles.
Calculate the speed of light in miles per second.

(SEG)

15. The mass, M , of the planet Mars is 6.45×10^{23} kg.
The planet is a sphere with radius, r , equal to 2.28×10^{11} m.
Use this formula to find its density:

$$\text{Density} = \frac{M}{\frac{4}{3}\pi r^3}$$

Express your answer in standard form, correct to three significant figures.

(SEG)

16. The surface of the Earth is approximately 1.971×10^8 square miles.
The surface area of the Earth covered by water is approximately
 1.395×10^8 square miles.

- (a) Calculate the surface area of the Earth not covered by water.
Give your answer in standard form.
- b) What percentage of the Earth's surface is not covered by water?

(SEG)

17. The mass of a neutron is 1.675×10^{-24} grams. Calculate the total mass of
1500 neutrons.

Give your answer in standard form.

(LON)